

**UNCLASSIFIED**

**AD 4 2 2 8 2 4**

**DEFENSE DOCUMENTATION CENTER**

**FOR**

**SCIENTIFIC AND TECHNICAL INFORMATION**

**CAMERON STATION, ALEXANDRIA, VIRGINIA**



**UNCLASSIFIED**

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

AD INU.  
DDC FILE COPY

422824

5739 200

1

NOV 15, 1963

5739 200

⑥ Analytical Approximations

Volume 5

⑩ by Cecil Hastings, Jr. and

J. P. Wong, Jr.

P-355 ✓

22 December 1952

The RAND Corporation

SANTA MONICA • CALIFORNIA

Copyright 1953

The RAND Corporation

## Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0017 over (0, 2),

$$q(2, x) \doteq .135 + .566 \left(\frac{x}{2}\right)^2 - .096 \left(\frac{x}{2}\right)^4 .$$

Cecil Hastings, Jr.  
James P. Wong, Jr.  
RAND Corporation  
Copyright 1952

12-11-52

### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p \, dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0008 over (0, 3),

$$q(3, x) \doteq .011 + .231 \left(\frac{x}{3}\right)^2 + .654 \left(\frac{x}{3}\right)^4 - .329 \left(\frac{x}{3}\right)^6.$$

Cecil Hastings, Jr.  
James P. Wong, Jr.  
RAND Corporation  
Copyright 1952

## Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p \, dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0011 over  $(1, \infty)$ ,

$$q(R, R) \doteq \frac{R + .183}{2R - .388} .$$

Cecil Hastings, Jr.  
James P. Wong, Jr.  
RAND Corporation  
Copyright 1952

12-18-52

### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p \, dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0004 over (0,1),

$$\begin{aligned} q(R, R) &= \frac{1}{2} \left[ 1 + e^{-R^2} I_0(R^2) \right] \\ &\doteq 1 - .4921R^2 + .3212R^4 - .0966R^6 . \end{aligned}$$

Cecil Hastings, Jr.  
James P. Wong, Jr.  
RAND Corporation  
Copyright 1952



12-20-52

### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0013 over (0,4),

$$q(4, 4-y) \doteq \frac{.551}{[1 + .187y + .055y^2 + .051y^3]^4}$$

Cecil Hastings, Jr.  
James P. Wong, Jr.  
RAND Corporation  
Copyright 1952